B.Sc Part I (Honours) Paper I

Example of Descartes's Rule of Signs

Q-1. Using Descartes rule of sign, find the nature of the roots of equation $x^4+15x^2+7x-11=0$

Ans – The given equation is of 4th degree so it has four roots

The signs of the terms of the polynomial are (+ + + –). Here only one change.

So the given equation can not have more than one positive root.

Further if $f(x) = x^4+15x^2+7x-11=0$

then $f(-x) = x^4 + 15x^2 - 7x - 11 = 0$

i.e. it has only one change of sign so $f(x)=0$ can not have more than one negative root

Also $f(-\infty) = +ve$, $f(0) = -ve$, $f(\infty) = +ve$

Here we observe that it has at least one negative root and at least one positive root.

Hence we can say that the given equation has one positive and one negative root.

Q-2. Show that the equation $x^5+x^3-8x+5=0$ cannot have more than three real roots and prove that it must have three real roots.

Ans:– let $f(x) = x^5+x^3-8x-5=0$

(+ + – –) i.e. only one change of sign in $f(x)$ hence $f(x)=0$ cannot have more than one positive root.

Also $f(-x) = -x^5-x^3+8x-5=0$

(– – + –) i.e. two change of sign in $f(-x)$

Hence $f(x)=0$ cannot have more than two negative roots.

i.e. $f(x)=0$ cannot have more than one positive and two negative roots

So $f(x)=0$ cannot have more than three real roots.

Further $f(-\infty) = -ve$ $\quad f(-2) = -29(-ve)$

$f(-1) = +ve$ $\quad f(0) = -5(-ve)$

$f(1) = -11(-ve)$ $\quad f(2) = 19(+ve)$

$f(\infty) = +ve$

i.e. $f(x)=0$ has roots between -2 and -1, where one root between -1 and 0 and other root between 1 and 2

Thus $f(x)=0$ must have three real roots in which two negative and one positive.

Q-3. Prove that $x^8+10x^3+x-4=0$ has at least six imaginary roots.

Ans – Let $f(x) = x^8+10x^3+x-4=0 \; (+++-)$

i.e. one change of sign.

Thus the given equation can not have more than one positive root.

Also $f(-x) = x^8-10x^3-x-4=0$

$(+---)$ i.e. only one change of sign

i.e. $f(x)=0$ cannot have more than one negative root.

But $f(x)=0$ has changes 8, hence it will have a total of 8 roots either real or imaginary or both.

Thus the minimum number of imaginary roots $= 8-1-1 = 6$.

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